Abstract

The importance of aerodynamics while climbing moderate hills is all too often underestimated. Yet the power required to overcome wind resistance increases as the cube of speed, whereas the power required to overcome the total weight of HPV and rider on a hill only increases linearly with speed. Because of this, a reasonably strong rider of a heavy but sufficiently aerodynamic HPV can climb a 3.5% grade faster than a rider of equal strength and weight riding a much lighter conventional racing bicycle. Five HPVs differing by as much as a factor of four in weight and a factor of nearly 28 in drag area (C_dA) are compared on grades between 3% and 5%. In the process, surprising equivalent efficiency nodes shared by three of the five vehicles emerge.

Aerodynamics and Weight on Moderate Hills

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At what uphill gradient does the low combined weight of the lightest conventional racing bicycle and its rider become more important than the aerodynamic advantage of a well designed human powered vehicle (HPV)? Primary considerations are obviously the weight and power output of the rider. A rider who is not especially strong may incorrectly assume that weight is everything on all hills. Certainly, if the rider is significantly overweight, out of shape, and riding a heavy HPV, an equally unconditioned but trim rider on a good lightweight conventional racing bike is probably going to climb faster on almost any perceptible upgrade, even if the heavy rider/HPV combination is somewhat more aerodynamic than the conventional rider and bicycle. Even rolling resistance is an important factor in this case, and it will be lower for the lightweight rider of the conventional racing bike. This is because rolling resistance is proportional to the total weight of rider and bike.

Things become more interesting if we compare riders of equal weight and of equal moderate to high strength using five diverse examples of competitive two-wheel HPVs. Let's assume the clothed, shoed, and helmeted riders of all five bikes each weigh 73.5 kg (162 lb). The gold standard for climbing is obviously the lightweight diamond frame drop bar road racing bicycle, which in 2008 must still conform to strict 1914 and 1934 *Union Cycliste Internationale* (UCI) retro-design requirements. All aerodynamic "devices" were banned in 1914, with recumbent and small-wheel designs subsequently outlawed 20 years later.[1] Current UCI bicycles with the 1934-mandated diamond frame/700C wheel configuration may weigh as little as 6.8 kg (15 lb), and on a suitable road surface might have a coefficient of rolling resistance, C_{rr}, as low as 0.004. Riding on the drops, a highly trained rider may be able to maintain a total frontal area, A, of just 0.33 m^2 with a coefficient of drag, C_d, as low as 0.89, for a drag area (C_dA) of 0.294 m^2.

An outstanding example of a truly modern, supremely comfortable dual suspension faired recumbent bicycle designed for long distance road racing and proven to excel in such competition[2] is one which I just happen to currently own myself: the Lightning F-90.[3] Weighing as much as two UCI racing bikes at 13.6 kg (30 lb) and using a 17 inch diameter (369 mm ISO-ETRTO) front wheel, it has a rolling resistance coefficient of about 0.006 on an average road surface. The builder, Tim Brummer, specifies a frontal area of 0.38 m^2 and a C_d of 0.38 for this machine with an aero-helmeted rider, yielding a C_dA of 0.144 m^2. Though its aerodynamic drag is less than half that of the UCI racer, its 6.8 kg (15 lb) weight handicap would seem to put it at a disadvantage for climbing grades as steep as 4%. We'll find out how much power an F-90 rider must crank out to hold his or her own with an equally strong UCI bike rider of equal weight on such a grade.

The third comparison HPV is another well known and proven, albeit extreme, streamlined recumbent design. The Varna Diablo,[4] designed and built by George Georgiev, currently holds the IHPVA straight-line high altitude unpaced speed record of 132.497 kph (82.33 mph), set Sept. 18, 2008, and the 2007 standing start unpaced World Hour record of 86.771 km (53.917 miles). Weighing double again at 27.2 kg (60 lb), it races with an estimated coefficient of rolling resistance of about 0.005. Its frontal area is specified to be a mere 0.191 m^2 and the C_d an amazing 0.097, from which a phenomenally low C_dA of 0.01853 m^2 is obtained! The question, then, is how this heavy but exceptionally aerodynamic recumbent bicycle might theoretically compare with the other two much lighter bikes for climbing moderate grades. The answer may surprise you.

Two other bicycles were included in the comparison calculations. The Lightning F-40[5] is similar in many ways to the Lightning F-90, but is about 3 kg (6.6 lb) heavier, a little less aerodynamic, but much more affordable. A lightweight example of the popular Sport Hybrid type of upright bicycle is included also, though not competitive in speed with the other machines. Table 1 contains relevant vehicle parameters for the five vehicles of interest. For each vehicle, the assumed rider mass = 73.5 kg (162 lb) and air density = 1.204945 kg/m^3.

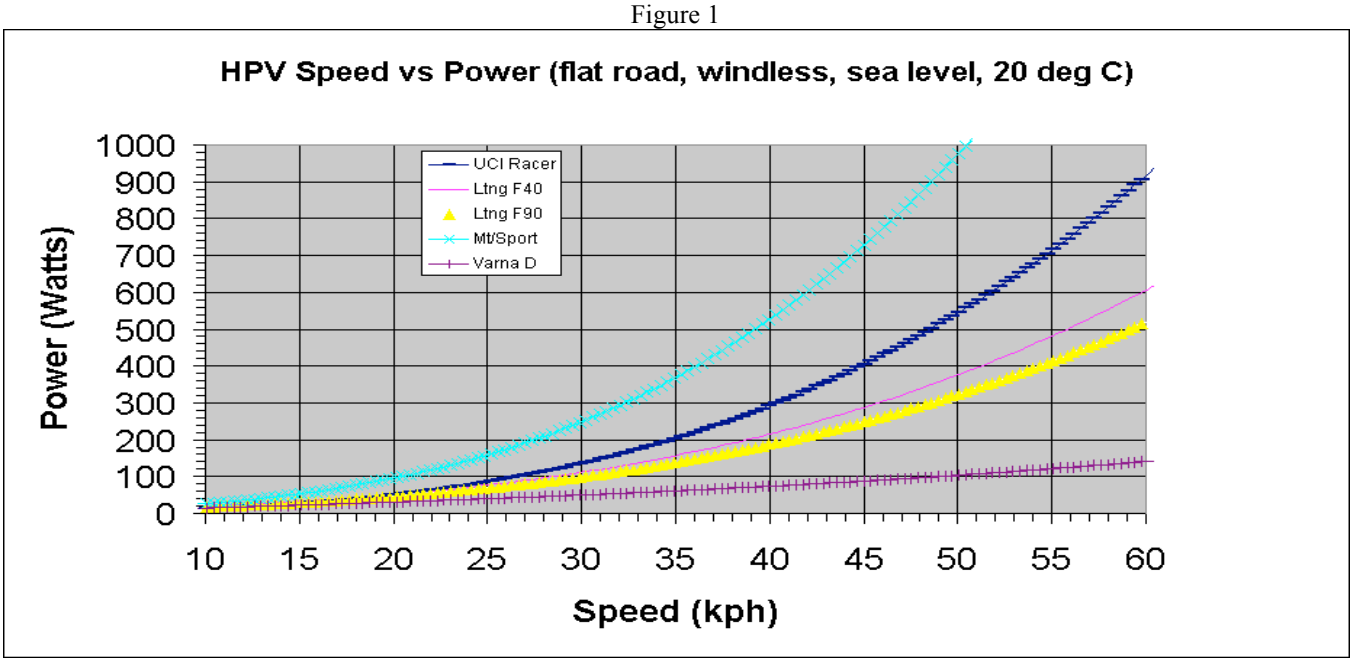
Table 1. HPV Parameters

Parameter	UCI drop bar racer	Lightning F-90	Varna Diablo	Lightning F-40	Sport Hybrid
M kg (lb)	6.8 (15)	13.6 (30)	27.2 (60)	16.6 (36.6)	11.5 (25.4)
C_dA (m^2)	0.294	0.144	0.01853	0.172	0.517
C_{rr}	0.004	0.006	0.005	0.006	0.008

Before comparing climbing capabilities on moderate upgrades, it is instructive to examine comparative power requirements as a function of speed on a perfectly level surface. For this, we compute the required power at the pedals in Watts as
$W=(a+b)*v/E_m$,
where
$a=C_dA*D*(v+w)^2/2$, the aerodynamic drag force in Newtons,
C_dA=drag area, in m^2,
D=air density (1.204945 kg/m^3 in this comparison). Based on the Standard Atmosphere model, sea level, 20° C (68° F),
v=velocity in m/s (kph/3.6),
w=head wind in m/s, zero for these comparisons,
$b=C_{rr}*g*M_t$, the rolling resistance force in Newtons,
Crr=coefficient of rolling resistance of vehicle tires,
g=acceleration of gravity, 9.806 m/s^2,
M_t=total mass of vehicle and rider. A 73.5 kg (162 lb) rider was assumed in calculations,
E_m=drivetrain mechanical efficiency (nominally 0.95. 0.94 used for F-40 and F-90, which have drive side idlers).

Figure 1 plots the predicted power (force times velocity) required as a function of velocity on a level road in windless conditions for the five vehicles of interest. The much higher speeds of the more aerodynamic HPVs are due to the fact that the power required to overcome air resistance increases as the cube of speed. Note that the UCI racer must expend 546 Watts to maintain 50 kph (31 mph) at sea level with no tailwind. This level of effort can be sustained for perhaps three minutes maximum by a strong racer.[6] Using aero bars and a disc rear wheel would allow the racer to sustain this speed with a significantly lower level of effort, and hence for a few more minutes, but in this analysis we only consider UCI road racing bicycles with no aerodynamic accessories that add weight, such as aero bars or disc wheels. At 50 kph, the Lightning F-90 rider expends just 324 Watts on a level, windless road at sea level. Because of the Lightning's dramatically lower drag area, this is a mere 59% of the power required of the UCI racer for the same constant speed. A well conditioned racer can produce this power continuously for about 90 minutes. But even an ordinary non-racer can produce the 104 Watts required to maintain 50 kph on a level road in Varna Diablo for several hours, provided that he or she can avoid overheating--a very real consideration with this vehicle.

Figure 1



At this point in the discussion, we are ready to calculate some theoretical climbing speeds on moderate upgrades for our five vehicles of interest. To do this, we must incorporate the force of gravity acting against objects on an inclined plane. The b term now becomes:

$b=M_t*g*(C_{rr}*\cos(\arctan(G/100))+\sin(\arctan(G/100)))$,

where

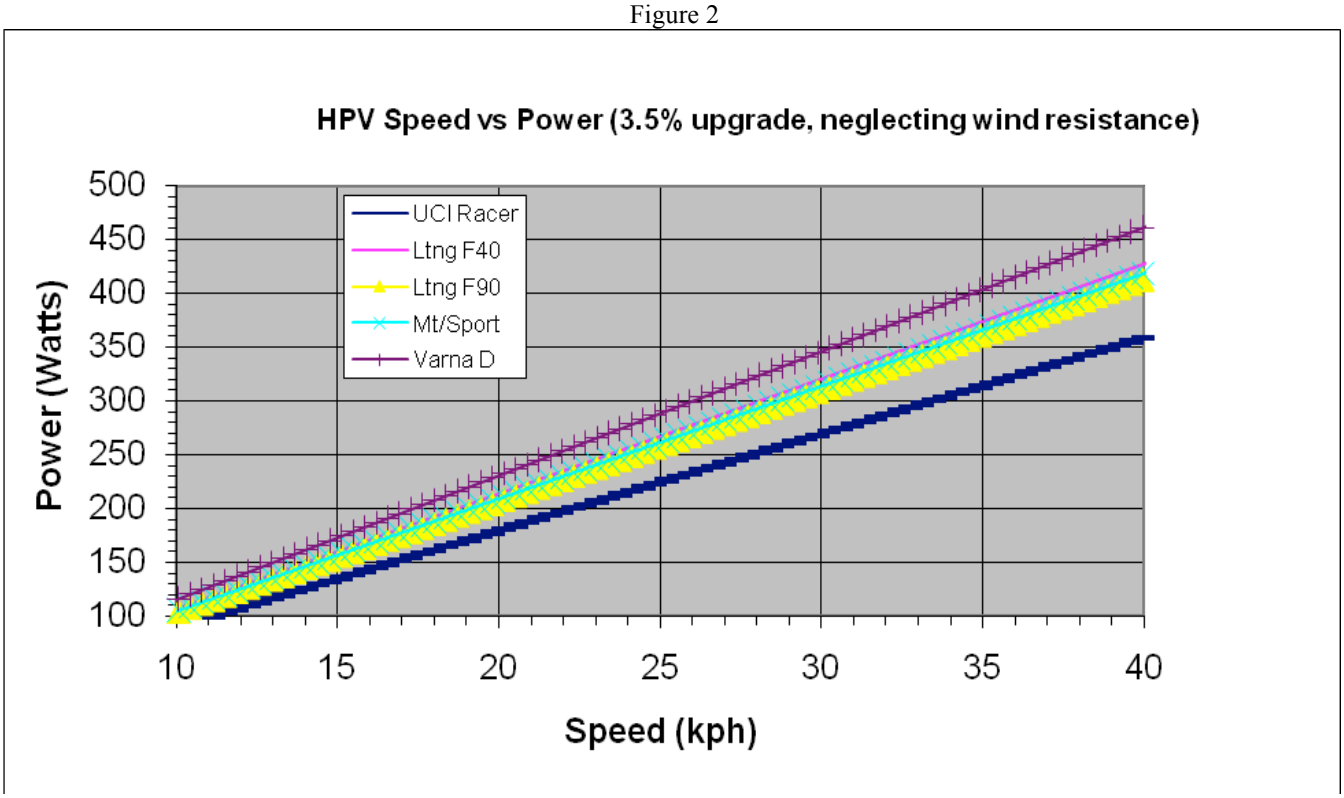
G = % grade.

An exactly equivalent algebraic form of the inclined plane equation, with no trigonometric functions, is

$b= M_t*g*(C_{rr}+G/100)/\mathrm{sqrt}(1+(G/100)^2)$.

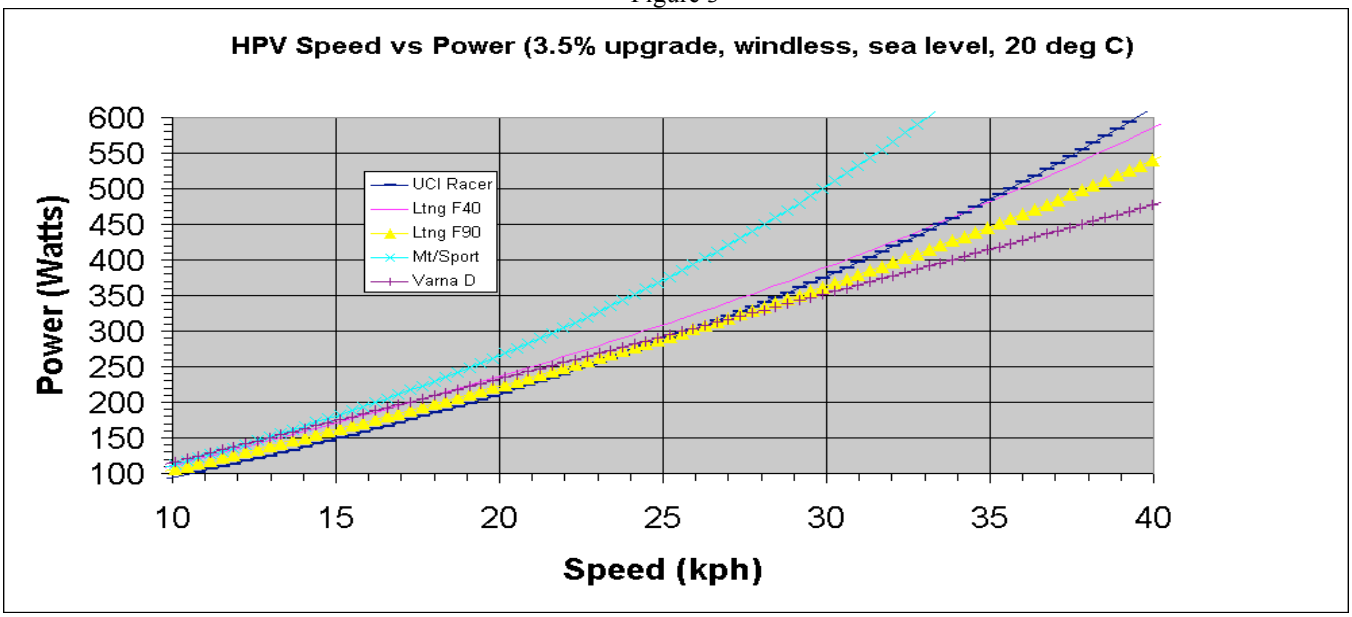
For descending grades, G is negative. The sin(arctan(G/100)) gravity term then becomes negative, denoting power supplied by gravity. The original equation for power (W) is accordingly not divided by E_m when coasting on downhills. Note that when b is multiplied by v as before, the combined forces of rolling resistance and gravity are converted to Watts of power, increasing linearly with speed on a given upgrade. Figure 2 shows this linear relationship between total mass and speed with respect to the power to overcome gravity and rolling resistance on a 3.5% upgrade, neglecting the effect of air resistance.

Figure 2



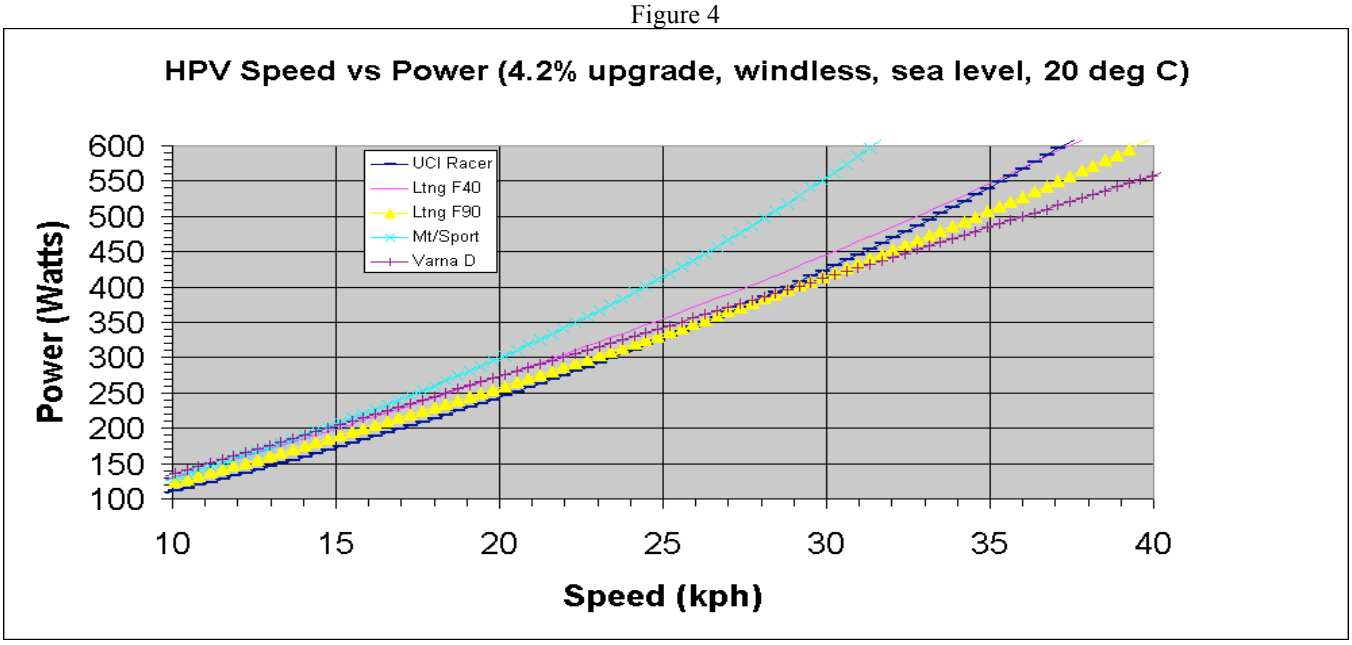
Next, Figure 3 plots the predicted total power requirements for each of the five vehicles to climb a 3.5% grade at various speeds. Here the power to overcome air resistance is added to the power to overcome gravity and rolling resistance. We are again reminded that the power required to overcome air resistance increases as the cube of speed while the required power to overcome gravity and rolling resistance, as seen in Figure 2, is merely directly proportional to total mass and to speed. Figure 3 shows an unanticipated shared nodal point of efficiency for the UCI, F-90, and Varna vehicles. We see that the curves for all three vehicles actually intersect at a single speed, 26 kph (16.2 mph), at which the power output of all three riders is also the same--300 Watts. This power level can be maintained for several hours by strong racers. What happens if the riders stop loafing and increase their power outputs to 350 Watts each? Figure 3 shows that to the right of the intersection point, the UCI rider can now climb this grade at 28.7 kph (17.8 mph), provided that an aerodynamic crouch is maintained. But the Lightning F-90 rider moves out at 29.1 kph (18.1 mph) at this power output and the 27.2 kg (60 lb) Varna virtually breaks away at 29.7 kph (18.4 mph)! Though four times the weight of the UCI bicycle, this superbly streamlined HPV wins the 3.5% grade climbing contest hands down, provided that its rider can produce 350 Watts for long enough to climb the hill!

Figure 3



In Figure 4, we steepen the upgrade to 4.2%. Interestingly, we again see a common nodal point of efficiency, but now the UCI, F-90, and Varna curves intersect at 28 kph (17.4 mph) and 384 Watts. Once again, if the riders are able to crank out even more power long enough to climb the hill, we see to the right of the intersection point that the heavy Varna Diablo climbs fastest, followed by the Lightning F-90, with the UCI bike dropping back.

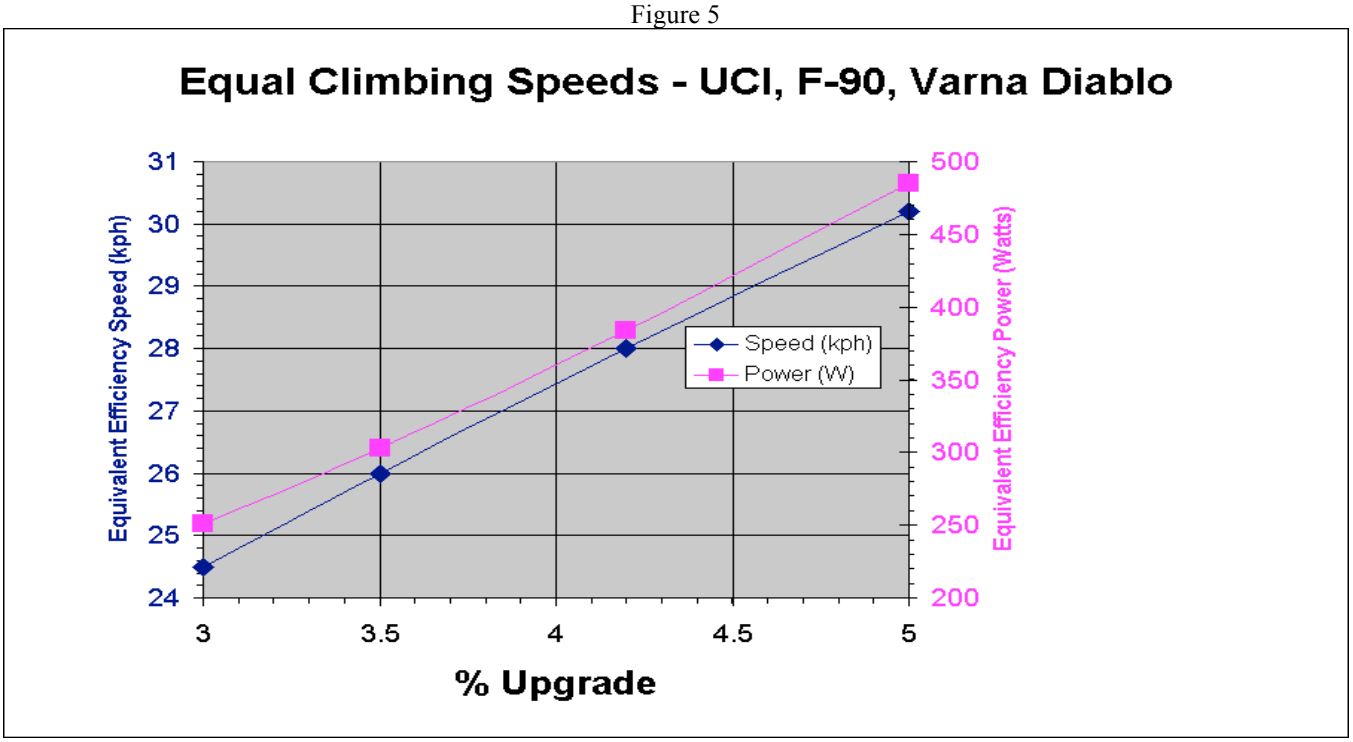
Figure 4



At this point you are probably wondering what range of upgrades exists within which all three fastest climbing bikes can be predicted to share common nodal points of efficiency at which equal speed is produced for equal power at the pedals. I too was curious, and found that such common points exist between 3% and 5% upgrades. The differences in speed among the three vehicles at each of the points of approximate intersection is small, less than 1% in the worst case of a 5% upgrade. These differences are most likely less than simulation model differences from actual vehicle performances in the real world.

Figure 5 plots the nodal points of equivalent efficiency for which approximately equal speeds for equal power are predicted for uphill gradients between 3% to 5%. We see that these speeds vary nearly linearly as a function of uphill grade, and the corresponding power plot is also only slightly nonlinear. It seems surprising that the combination of the three very different aerodynamic drag forces, which each increase as the square of velocity, and the three different combined gravitational and rolling resistance forces, which are each constant, conspire to produce a single near-linear velocity function from a nearly linear power function. In this range of grades, an increase in power to the area above the red power line always produces the greatest increase in speed from the Varna Diablo and the second greatest from the Lightning F-90, with the UCI bike consistently attaining the slowest steady state velocity from the increased power. Again, this is because the power to overcome aerodynamic drag increases as the cube of velocity, but the power to climb against gravity and to overcome rolling resistance only increases proportional to speed. The combined vehicle and rider weights vary by a factor of 1.254 from the lightest to the heaviest, whereas the drag areas vary by a factor of 15.866 in the opposite direction among the three vehicles of interest. The total range of power required to produce the range of resulting plotted equal speeds for the three vehicles is 234 Watts, from 250 Watts on a 3% grade to 484 Watts on a 5% grade. The equivalent efficiency speeds plotted range from 24.5 kph (15.2 mph) on a 3% grade to 30.2 kph (18.8 mph) on a 5% grade.

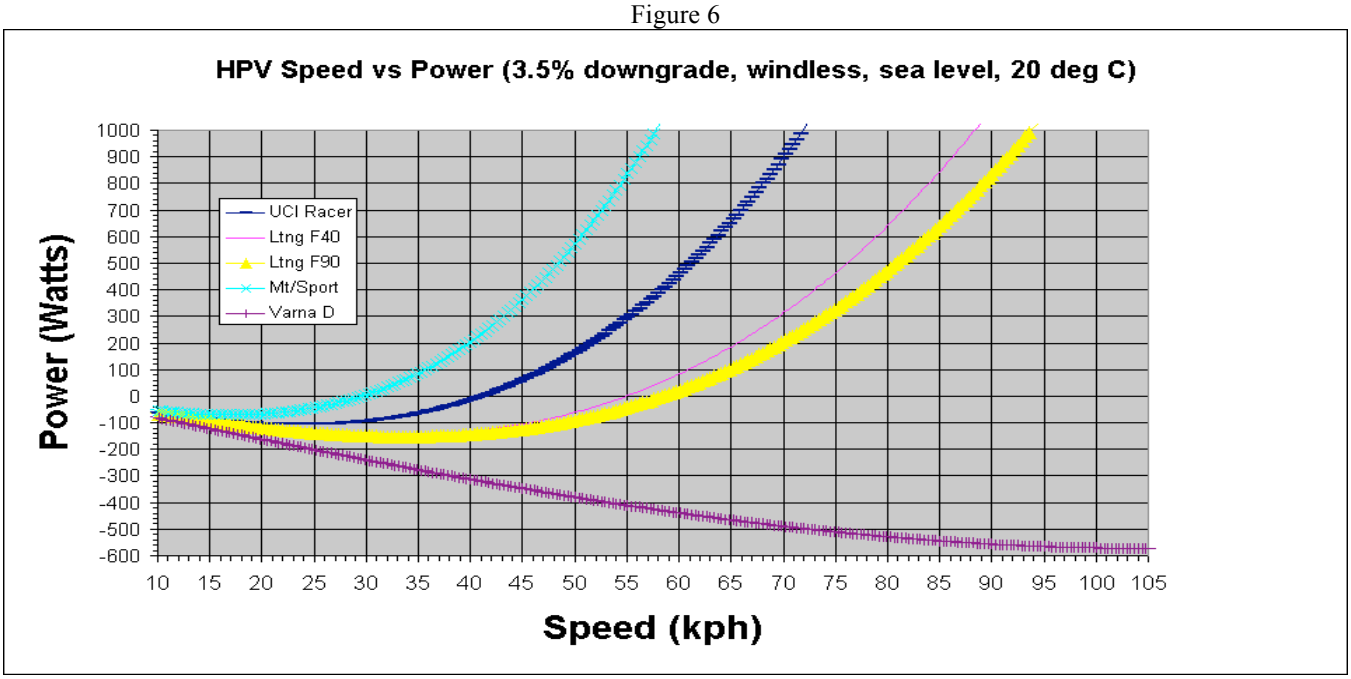
Figure 5



Up to now we have been discussing constant climbing speeds on constant grades. In practice, a very significant advantage is obtained by the heavier vehicle/rider combinations through being able to store momentum from a short level stretch before the foot of the hill. For example, if the bicycles approach the base of the hill at 50 kph (13.89 m/s), the momentum, m*v, of the Varna and rider is 1399 kg-m/s. It is 1210 kg-m/s for the F-90 and rider, and 1115 kg-m/s for the UCI bike and rider. In terms of kinetic energy, $m*v^2/2$, the Varna and rider store 9714 Joules, the F-90 and rider 8402 Joules, and the UCI bike and rider 7746 Joules. A Watt is one Joule per second, so the Varna rider effectively converts his 1968 surplus Joules over the UCI rider into an extra free 100 Watts for 20 seconds after starting up the hill. The Lightning F-90 rider will effectively obtain 50 free Watts more than the UCI rider for 13 seconds. The effect of momentum is to give the heavier, more aerodynamic HPVs a surprisingly large advantage in rolling hills.

The 3.5% downgrade case is instructive. On this moderate downgrade, the heavier and more aerodynamic vehicles have an overwhelming terminal velocity advantage over the UCI bicycle. Figure 6 compares downhill speeds on a 3.5% downgrade. It is seen that Varna Diablo reaches maximum gravity assist power at 104 kph (64.6 mph). Varna's off-scale coasting terminal velocity is 180 kph (112 mph)! By contrast, the F-90 rider has to exert 303 Watts at the pedals to maintain 74 kph (46 mph) and the UCI rider can only maintain 55.2 kph (34.3 mph) with the same power output.

Figure 6



In my favorite scenario, a strong young UCI racer is halfway up a 4% hill when passed by an elderly Lightning F-90 or F-40 rider moving considerably faster. The UCI cyclist suspects that the Lightning fairing is concealing an electric motor or that the Lightning rider is exceptionally strong for his age. What is actually happening is that the UCI racer has slowed down to his steady state climbing speed, whereas the Lightning rider is still well above that speed because of his extra momentum from the previous downhill. In the most entertaining scenario, the Lightning rider crests the hill just as the UCI rider almost overtakes. If the hill is longer, the strong young UCI racer will have the brief pleasure of passing the elderly Lightning rider before the crest, but the subsequent downhill will soon turn the tables, perhaps halfway up the next hill.

How did the myth that "recumbents can't climb" take hold so strongly? On very steep grades for which climbing speeds are too low for aerodynamics to be a significant factor, it is true that conventional bikes have a two-fold advantage. Light weight is the most obvious one. Lance Armstrong's Tour de France climbing bike weighed 6.8 kg (15 lb). One of the lightest racing/touring recumbents made is the 8.6 kg (19 lb) Lightning R-84, which is essentially a Lightning F-90 without the fairings. The 1.8 kg (4 lb) of extra weight over Lance's factory works Trek is somewhat of a disadvantage on steep grades.

Also, the conventional design typically allows better use of the back and arm muscles while climbing. Standing on the pedals of a conventional bike is thought by many to allow more power to be transmitted to the rear wheel, but this is just another myth, since there is no such thing as perpetual motion. For every application of weight to a pedal, that weight must be lifted against gravity before the next stroke. However, a strong recumbent rider can theoretically push against the seat back with more force than his or her weight, which is arguably an advantage for the recumbent rider. In practice, the fact that the typical recumbent design does not favor using the handlebars for utilizing upper body muscles seems to be more important than the ability to support the back while pushing forward with the feet. Positioning the handlebars more forward, and making the long stem post sturdy enough to ensure it will not break could help, although this can only add more unwanted weight.

The real reason, however, that recumbents are seen as poor climbers even on moderate grades lies with the type of people who buy and ride these bikes and trikes. The vast majority of people who ride recumbents do so because of their superior comfort, not speed. Because of this, they are correctly perceived as not being athletic or competitive riders. Competitive bike riders have an obvious rational reason for using less comfortable, less efficient conventional bikes--the UCI rules forbid the use of recumbents in major competition. The 2004 Race Across America was an exceptional, even historic, event in this regard. It was the first major bicycle race since 1934 in which recumbent bicycles were allowed to compete head to head with conventional bicycles. Yet riders of traditional bikes are still loathe to acknowledge the superior road performance of aerodynamic recumbents, even after such an overwhelming victory as the Lightning Team's in the 2004 RAAM. Long held traditional beliefs are very slow to change and experts don't like to admit that their preconceived notions can be dead wrong.

Here in Southern California, several strong riders of conventional racing bikes who use a local beach bike path have already learned that at least one recumbent (my Lightning F-90) can climb better than they can on their traditional racing bike on the little hill ascending to our hang glider launch area. This newfound knowledge is visibly upsetting to them, but it is slowly resulting in a long overdue acknowledgement of the superiority of the recumbent design.

-Gerry Pease

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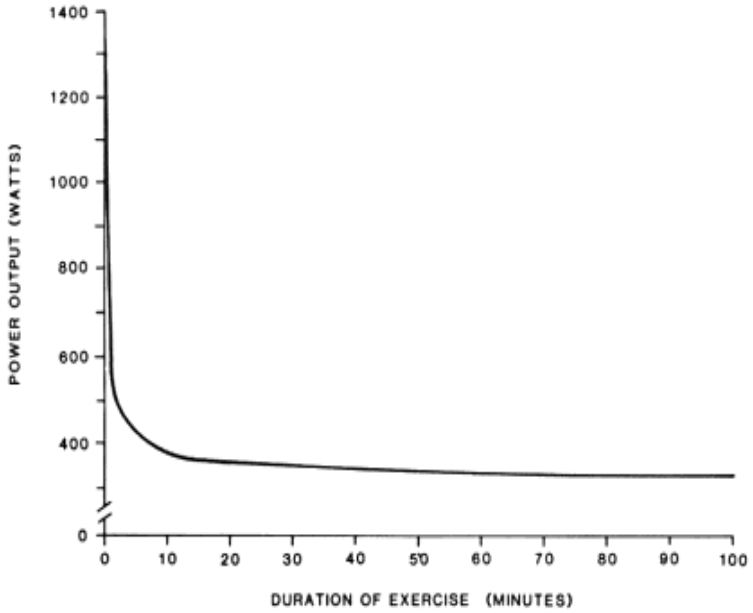
6 http://www.mne.psu.edu/simpson/courses/me288/ME288.bike2.ppt - slide 16, reproduced below:

Human Power Output



The maximum power output that can be sustained for various time durations by strong racing cyclists. Average power output over long distances less than 400 W.